# Facilitating Communities of Mathematical Inquiry

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In the current shifts in mathematics classrooms teachers are challenged to use effective pedagogy to develop inquiry communities in which all participants are offered opportunities to engage in the reasoning discourse of proficient mathematical practices. The challenge for teachers is to know what pedagogical actions support the development and use of effective mathematical practices. This paper examines how a group of teachers used a purposely designed communication and participation framework as a tool to scaffold development of inquiry communities and the use of progressively more proficient mathematical practices within their classrooms..

The past decade has seen ambitious calls by national and international policy makers and researchers for significant changes to be made to the teaching and learning of mathematics (e.g., Franke, Kazemi, & Battey, 2007; Ministry of Education, 2007; RAND, 2003). Increasingly, reform efforts have centred on the potential benefits of giving specific attention to the teaching and learning of mathematical practices; practices which encompass the mathematical know-how which constitutes expertise in learning and using mathematics (Boaler, 2003b; RAND). The RAND group maintain that for students to develop robust mathematical thinking and reasoning processes students need opportunities not only to construct a broad base of conceptual knowledge; they also require ways to build their understanding of mathematical practices. A common theme of literature related to development of mathematical practices (Ball & Bass, 2003; Boaler, 2003b; Franke et al., 2007) is the need for students to participate in reasoned collective discourse so that they learn to construct and communicate powerful, connected, and well reasoned mathematical understanding. In keeping with this notion, Ball and Bass explain that reasoning in this form "comprises a set of practices and norms that are collective … and rooted in the discipline" (p. 29).

The advocated changes necessitate radically different roles and responsibilities for the teachers and students, including how they are to relate to each other, to the classroom power and authority base, and to the discipline of mathematics itself (Boaler, 2003a; Sowder, 2007). A central hallmark of the changes is a vision of students and teachers actively engaged in the shared dialogue of inquiry and argumentation, using the mathematical practices of able problem solvers, within classrooms which resemble learning communities (Goos, 2004). For example, in the new national curriculum document for New Zealand, teachers are charged with the responsibility of putting into effect pedagogy which facilitates inquiry climates where "everyone, including the teacher, is a learner; learning conversations and learning partnerships are encouraged; and challenge, support, and feedback are always available" (Ministry of Education, 2007, p. 34). Here the mathematical practices include "justifying claims, using symbolic notation efficiently, defining terms precisely, and making generalisations [or] the way in which mathematics users are able to model a situation to make it easier to understand and to solve problems related to it" (RAND, 2003, p. xviii).

Of central importance in shifting the discourse towards inquiry are the enacted sociocultural and mathematical norms (Sullivan, Zevenbergen, & Mousley, 2002); the negotiated variables constituted within discursive interaction. Sociocultural norms relate to the stable patterns of behaviour or practices, organisational routines, and forms of communication valued in classroom communities. Mathematical norms support higher level cognitive activity and relate directly to mathematics. They evolve within mathematical activity and are the "principles, generalisations, processes and products that form the basis of the mathematics curriculum and serve as the tools for the teaching and learning of mathematics itself" (Sullivan et al., p. 650). Theorising that mathematical norms and mathematical practices are interrelated offers a way to explain how mathematical practices are transformed as they are negotiated in the discursive dialogue and emphasises the importance of teachers attending to the discourse used in mathematical activity. A number of studies (e.g., Franke et al., 2007; Hufferd-Ackles, Fuson, & Sherin, 2004; Wood & McNeal, 2003) have illustrated deeper student engagement in mathematical practices when teachers explicitly foster communicative patterns which shift the collective discourse from inquiry to argumentation, challenge, and debate.

Successful implementation of such discourse is a challenging task. The literature provides ample evidence (e.g., Franke et al., 2007; Hufferd-Ackles et al., 2004) of the considerable complexities involved in establishing collective discourse which provides students with space to engage in disciplined ways of reasoning and inquiry. For many teachers their fundamental beliefs about teaching and learning are challenged as they rethink their roles and responsibilities and those of their students within the classroom discourse patterns. At the same time, the changed communication and participation patterns also create challenges for students. Not only is their notion of the teacher's role as unquestionable authority in dispute; changes reflecting the wider diversity of their roles, task demands, and novel interactional scripts also add to the demand (Forman, 1996).

Whilst readily acknowledged that the pedagogical actions used to guide and negotiate the mathematical and sociocultural norms are pivotal to facilitating communities of mathematical inquiry, it is less clear how teachers might effect such a change. This is particularly an issue for those teachers currently in the classroom who too often lack experience of learning in inquiry environments or using effective mathematical practices (Hufferd-Ackles et al., 2004; RAND, 2003). Seldom do curriculum documents clarify a teacher's role in such learning environments, nor provide guidance on how to constitute the sociocultural and mathematical norms (Sullivan et al., 2002). In addressing this concern, this paper reports on how a group of teachers used a purposefully designed communication and participation framework to map out the establishment of an inquiry community. The central focus of the paper is on how the teachers adapted and used the framework as a tool to constitute the sociocultural and mathematical norms of mathematical inquiry communities. Exemplars of how the teachers scaffolded student engagement in reasoned discourse that supported the use of more proficient mathematical practices capture the ever present "dynamic process of interpretation and mutual adjustment that shapes students learning" (Ball & Forzani, 2007, p. 531) within inquiry communities.

The theoretical standpoint of this study is derived from a sociocultural perspective on learning. Within this perspective, mathematics learning is viewed as contextualised, which is to view learning-in-activity. The social, cultural, and institutionalised contexts are not considered merely as factors which may aid or impede learning; rather, these social organisational processes are integral features of the learning itself and are mutually constitutive (Forman, 1996). As Lerman (2001) explains, when the social practices of classroom communities are discursively constituted "people become part of practices as practices become part of them" (p. 88). Thus, within the sociocultural lens of this study, the learning and use of mathematical practices is matched by an "increasing participation in communities of practice" (Lave & Wenger, 1991, p. 41); a dynamic process of change which involves shifts in positioning of all members of the community.

## Research Design

The reported research is from a larger classroom-based design study (Hunter, 2007). Conducted at a New Zealand urban primary school, the study involved four teachers and 120 Year 4-8 students. The majority of students came from low socio-economic home environments and were of Pasifika or New Zealand Maori ethnic groupings.

A year-long partnership between researcher and teachers supported the design and use of a participation and communication framework (see Table 1). Adapted from the theoretical framework proposed by Wood and McNeal (2003), the framework drew on a wide range of research findings related to those communication and participation patterns that had been found to be effective in supporting student engagement in a variety of mathematical practices. These were positioned in the framework as conjectures of possible actions teachers could scaffold students to use, to provide them with opportunities to learn and use mathematical practices within the collective inquiry discourse.

As an organising tool to assist teachers to scaffold students' use of proficient mathematical practices within reasoned inquiry and argumentation, the framework was structured around two components: communication patterns and participation patterns. Vertically, the framework outlined a set of collective reasoning practices matched with conjectures relating to the communicative and performative actions teachers might require of their students to scaffold their participation in learning and using mathematical practices. Likewise, conjectures of a set of participatory actions teachers may expect of their students to promote their individual and collaborative responsibility in the collective activity were included. The horizontal flow over three phases sketched out a possible sequence of communicative (and performative) and participatory actions teachers could scaffold their students to use as they went about establishing communities of mathematical inquiry.

**Table 1**

*The Communication and Participation Framework*

| | Phase One | Phase Two | Phase Three |
|---|---|---|---|
| Making conceptual explanations | Use problem context to make explanation experientially real. | Provide alternative ways to explain solution strategies. | Revise, extend, or elaborate on sections of explanations. |
| Making explanatory justification | Indicate agreement or disagreement with an explanation. | Provide mathematical reasons for agreeing or disagreeing with solution strategy. Justify using other explanations. | Validate reasoning using own means. Resolve disagreement by discussing viability of various solution strategies. |
| Making generalisations | Look for patterns and connections. Compare and contrast own reasoning with that used by others. | Make comparisons and explain the differences and similarities between solution strategies. Explain number properties, relationships. | Analyse and make comparisons between explanations that are different, efficient, sophisticated. Provide further examples for number patterns, number relations and number properties. |
| Using represent-ations | Discuss and use a range of representations to support explanations. | Describe inscriptions used, to explain and justify conceptually as actions on quantities, not manipulation of symbols. | Interpret inscriptions used by others and contrast with own. Translate across representations to clarify and justify reasoning. |
| Using mathematical language and definitions | Use mathematical words to describe actions. | Use correct mathematical terms. Ask questions to clarify terms and actions. | Use mathematical words to describe actions. Reword or re-explain mathematical terms and solution strategies. Use other examples to illustrate. |

| | | | |
|---|---|---|---|
| Participatory actions | Active listening and questioning for more information.<br><br>Collaborative support and responsibility for reasoning of all group members.<br><br>Discuss, interpret and reinterpret problems.<br><br>Agree on the construction of one solution strategy that all members can explain.<br><br>Indicate need to question during large group sharing.<br><br>Use questions which clarify specific sections of explanations or gain more information about an explanation. | Prepare a group explanation and justification collaboratively.<br><br>Prepare ways to re-explain or justify the selected group explanation.<br><br>Provide support for group members when explaining and justifying to the large group or when responding to questions and challenges.<br><br>Use wait-time as a think-time before answering or asking questions.<br><br>Indicate need to question and challenge.<br><br>Use questions which challenge an explanation mathematically and which draw justification.<br><br>Ask clarifying questions if representation and inscriptions or mathematical terms are not clear | Indicate need to question during and after explanations.<br><br>Ask a range of questions including those which draw justification and generalised models of problem situations, number patterns and properties.<br><br>Work together collaboratively in small groups examining and exploring all group members reasoning.<br><br>Compare and contrast and select most proficient (that all members can understand, explain and justify). |

Data collection over one year included three teacher interviews, twice weekly video captured lessons, field notes, classroom artefacts, written and recorded teacher reflective statements, and teacher recorded reflective analysis of video excerpts. On-going data collection and analysis maintained focus and flexible revision of the emerging communication and participation patterns which supported development of proficient mathematical practices. Data analysis occurred chronologically using a grounded approach to create codes, and to identify coherent patterns and themes. Trustworthiness was maintained through use of constant comparative methods which involved interplay between the data and theory and sustained engagement with participants in the field by the researcher.

## Results and Discussion

The communication and participation framework was used by the teachers as a flexible and adaptive tool to map out development of an inquiry environment in which the students were offered opportunities to participate in learning and using progressively more proficient mathematical practices. In using the framework the teachers all reached similar endpoints at the conclusion of the study, but the individual pathway they each mapped out and traversed was unique. How the teachers positioned themselves in the mathematical discourse was an important initial consideration; those who positioned themselves in a more traditionally oriented role encountered more challenges in effecting change and their journey to construct a community of inquiry was lengthened.

### *Constituting Intellectual Partnerships which Supported Mathematical Arguments*

To initiate change, all of the teachers initially addressed the sociocultural norms in their classrooms. They repositioned themselves as participants in the discourse and they emphasised student responsibility for active listening and sense-making. Their immediate focus aimed to establish safe, supportive learning environments that promoted social and intellectual risk-taking. They employed a range of strategies to attend to students' affective needs, including direct discussion of the need for collegiality, inclusion, and intellectual and social risk-taking. Each of the teachers also implemented specific strategies to re-mediate situations in their existing classroom culture, including closely engineering learning partnerships (e.g., placing Maori, Pasifika and female students in supportive pairs initially), and the use of specific talk-formats that valued assertive communication, construction of multiple perspectives, and affirmation of effort over ability. For example, in the following teacher comment the social and cultural background of the students were drawn on and linked to the expectations and obligations of the developing community—referred to as a whanau—a family and collective concept in which the more knowledgeable are positioned as valued knowledge sources within the collective:

Teacher: Remember you are a member of our whanau so you need to be loud and proud and confident … we are all ready to think and listen.

A focus on communal construction and examination of mathematical explanations occurred in partnership with development of the sociocultural norms. Guided by the framework, the teachers addressed how students in small heterogeneous groups were to discuss, negotiate, and construct a collective solution strategy. However, each teacher adapted the communication and participation framework to guide the specific needs of their classroom context. For example, Table 2 illustrates an extended adaptation one teacher made to the trajectory she used within her class to help her students develop ways of managing their initial discussions.

**Table 2**

*An Adapted Section: Phase 1 Making Mathematical Explanations*

| | |
|---|---|
| Think of a strategy solution and then explain it to the group. Listen carefully and make sense of each explanation step by step.<br><br>Make a step by step explanation together. Make sure that everyone understands. Keep checking that they do.<br><br>Take turns explaining the solution strategy using a representation.<br><br>Use equipment, the story in the problem, a drawing or diagram or/and numbers to provide another way or backing for the explanation. | Keep asking questions until every section of the explanation is understood.<br><br>Be ready to state a lack of understanding and ask for the explanation to be explained in another way.<br><br>Ask questions (what did you…) of sections of the explanation.<br><br>Discuss the explanation and explore the bits which are more difficult to understand.<br><br>Discuss the questions the listeners might ask about the explanation |

In the sharing sessions which followed small group activity, student presentation and sense-making of conceptual explanations were closely structured. The teachers provided models of questions to elicit further clarification of the reasoning, and they prompted explainers to make the explanations experientially real for the listeners. They also directly interceded and structured the discourse to allow space and time for sense-making. As a result, within each classroom, within differing timeframes, the students realised their responsibility for reasoned sense-making. At the same time, the communal construction and examination of explanations as mathematical arguments provided an important foundation for the teachers to press towards explanatory justification and generalisation.

*Maintaining Intellectual Partnerships in Collective Justification and Generalisation*

The participation structure the teachers made available to the students operated as a scaffold for the development of argumentation. However, the shift to consider mathematical explanations as a form of argument caused conflict for the teachers. Not only did they acknowledge their own novice status in an inquiry environment, they also expressed concern at what they considered to be a lack of fit between the students' cultural and social norms and the requirement that they engage in the discourse of inquiry and argumentation.

Using the framework as a reflective tool the teachers critically analysed video excerpts for student engagement in interrelated mathematical practices. They examined student use of the questions and prompts which supported emergence of mathematical practices. They re-mapped their pathways and planned their next foci. Student attention was focused beyond the development of mathematical knowledge to rich ways to use and extend the reasoning mathematically. With their attention directed to discussing and modelling mathematical argumentation, the teachers required that the students construct multiple explanations. The teachers used rich tasks and problems they had collaboratively designed in accord with their next goals. To strengthen their ability to encounter challenge students were required to examine their arguments closely and rehearse possible responses to questions or challenge. The teachers scaffolded and probed the students to use the questions, and prompts which drew justification and generalisations. They promoted the use of "thinking time" as a pause in the dialogue to provide the students with opportunities to analyse explanations, frame questions, and reconsider and restructure arguments. They also explicitly positioned students to voice agreement or disagreement backed by mathematical reasoning.

The use of these practices provided the students with a predictable framework for strategy/solution reporting, inquiry and argument and resulted in extended reasoned dialogue. A consistent pattern occurred in each classroom; as the discourse of inquiry and argumentation increased the teachers began to explicitly focus on, attend to, and build on, the students' observations of patterns and relationships. This is illustrated in the following episode in which a teacher asks the students to analyse a strategy in which a student had justified his group's collective explanation for a decimal problem using fractions:

Sally: There's three different ways to basically explain a fraction, the fraction way, a decimal point way, and a decimal way. That's why he has picked one of them. Instead of just doing the fraction or percentage, he's picked the decimal point way because he may think that that is actually his easier point of doing the fraction way.

Teacher: But can you do that?

Sonny: Yes because they are equivalent like just the same.

The increased student agency in the discourse led to repositioning of all participants in the classrooms. Within the negotiated and extended dialogue the teachers assumed facilitative roles, stepping in and out of the dialogue as they guided development of a shared perspective from which all community members drew on a range of different mathematical practices as integrated tools for *using* and *doing* mathematics. The following excerpt illustrates how a teacher supported her students to autonomously use their mathematical knowledge and practices to analyse a solution strategy for a problem which involved multiplying forty by twenty-four:

Kuini: [Examining the representation] Hang on. So if you are saying you got the two from the twenty then do you mean that ten times twenty-four equals two hundred and twenty-four times another two equals four hundred and eighty is the same as twenty times twenty-four? But why start with two? You need to convince us.

Kuini has closely examined the representation and uses her interpretation to question further. The teacher without speaking turns to Akeriri and nods to affirm his need to provide explanatory justification.

Akeriri: Because two is easier than four timesing. It's sort of like what Saawan showed us yesterday. Yeah and then I go times two again and it's nine hundred and sixty because that is the same as four times ten times twenty-four or forty times twenty-four. Are you all convinced?

In the continued exploratory discussion within the discourse of mathematical inquiry and argumentation the students connect previous reasoning and explore new directions.

Pania: Hey. Would that strategy work with other numbers? Hey what about this? You could do eighty times over twenty-four hours.

Guided by the teacher's facilitative stance the students continue to explore and analyse other numbers and patterns, using agreement and disagreement validated by mathematical evidence.

Kuini: I agree but those are all even numbers. So does it work with only even numbers because you can't half an odd number?

Akeriri: Saawan did it yesterday when he did nineteen but that wasn't the same strategy, that's changing it.

Pania: I disagree. It's just changing the numbers. I think you could take one lot off and then multiply it and then add the one lot back and it's the same. Or use Akeriri's way put the other one back. It works so you can do odd.

Through the extended examination of the argument the reasoning is validated through use of explanatory justification and generalised reasoning.

## Conclusions and Implications

Although the teachers and their students all began as novices in the discourse of inquiry and argumentation the participation and communication framework provided a flexible tool which over the duration of the study supported the renegotiation of contexts and the development of detailed pathways for individual classroom communities. The attention placed on the sociocultural and mathematical norms was of significance in developing communal dialogue and individual and collective responsibility to sense-make. Success with scaffolding the students' participation in mathematical reasoning at higher intellectual levels in turn affirmed the teachers' continued press for inquiry and argumentation. In accord with current literature (e.g., Franke et al., 2007; RAND, 2003; Wood & McNeal, 2003) the teachers' increased expectations provided the students with a platform to learn and use explanatory justification, generalised reasoning, the construction of a range of inscriptions to validate the reasoning, and a more defined use of mathematical language.

The participation and communication framework was an effective tool which focused teachers' attention on specific communicative and performative actions they might require students to use to scaffold their engagement in the interrelated mathematical practices. Importantly in this study, the teachers, working within a supportive community of teacher learners including the researcher, were able to adopt and adapt the framework to meet their precise needs. Further research with different groupings of teachers and students to explore the adaptations that particular teachers make to the framework is needed. A greater understanding, both of the framework tool and the associated professional development, is needed to enable such a tool to be more widely used to support teacher learning and change.

## Practical Implications

The challenge of creating and sustaining change in teachers' pedagogical practices is a well-documented and on-going issue. Extant beliefs and attitudes teachers hold about their role in the mathematical discourse and activity of mathematics classrooms which shape how they position themselves can prove a significant barrier to change, as can teachers' prior experiences. Importantly many teachers have not experienced learning (or teaching) in classrooms which promote mathematical dialogue, inquiry and argumentation—nor in those which explicitly focus student learning beyond acquiring mathematical knowledge towards learning and using proficient mathematical practices to *do* and *use* the mathematical knowledge. Findings reported in this paper show how teachers can successfully be scaffolded to reflectively adopt and adapt new pedagogical practices that represent a shift from the traditional foci on rote learning of computational rules and procedures to one in which all members of the community are active participants in collective analysis and validation of mathematical reasoning.

In recent times most teachers in New Zealand primary schools have had opportunities to participate in national numeracy professional development programmes. However, how to sustain the initial teacher change and support generative teacher change remains challenging. Importantly, this study found the communication and participation framework (CPF) gave those teachers who had prior involvement in the Numeracy Project an effective tool to support substantive professional learning. The framework not only scaffolded teachers to critically examine their pedagogy and classroom practices in terms of inquiry but prompted ongoing generative change towards creating a more effective community of mathematical inquiry as evidenced by students' use of increasing sophisticated mathematical practices,

To enact substantive professional learning requires that the preconceptions, prior experiences, and practical routines—the tacitly held personal theories of action teachers hold—are interrupted. For change to current practices to occur teachers require space to consider, reflect on, and as appropriate experience dissonance in these routines. This paper suggests that within a study group setting with spaces for individual and collective reflection—the CPF potentially provides a useful and practical tool teachers can use to analyse and understand the tacit theories of action which underpin their current practices.

Critical to using the CPF was the teachers' involvement in study groups comprised of teachers/teachers and/or a researcher, and access to, and discussion of, research literature which provide models of mathematical practices in inquiry environments. By making direct links to the communicative and performative actions outlined within the first phase the teachers were able to critique and evaluate the adequacy of their currently enacted classroom sociocultural and mathematical norms and in turn map out possible change scenarios and pathways. The descriptive detail of communicative and participatory actions teachers may require students to use provided an overt way to interrogate current practice while also acting as a mentoring tool for independent or collegial planning. This was especially effective when applied to the video replay of the teachers' lessons. Such analysis, when guided by the CPF, provided key points for discussion and pressed teachers to develop their own sets of questions and prompts and rich mathematical tasks for engaging students in mathematical inquiry practices. Through collegial discussion, examination, exploration and trialling of the CPF the teachers were provided with opportunities to learn in the act of developing new ways to orchestrate classroom interaction patterns.

A key feature of the CPF design was the horizontal organisation of phases. This signalled to teachers that shifting between and across the phases and the mathematical practices to match the shifting needs of the classroom context was something that took time. As they themselves through their professional growth gradually shifted from novice to expert facilitators of the mathematical discourse and practices, so too would their students need time to acquire competence and confidence with a range of mathematical practices associated with mathematical inquiry. In practice, the immediate and continued focus on the development of sociocultural norms caused gradual shifts in the roles and responsibilities of all members of community. Within these important interactional shifts both the teachers and students had time and space to practise and explore using the developing discourse. Moreover, the space within the phases provided the teachers with opportunities to examine and explore appropriate ways they could draw on their students' home contexts to develop mathematical argumentation in socially and culturally responsive ways.

For change to be sustained beyond a professional development programme teachers need to both adopt and adapt new knowledge into their own conceptual framework about teaching and learning. Integral to the use of the CPF in this study was the opportunities for teachers to apply the information and skills within their own situated practice. Learning in the act of teaching is influenced by the space and time provided to them and the depth of their professional growth is related to their interaction with the new learning. For example, at the beginning of the study the participating teachers, influenced by their recent involvement in the Numeracy Project, included student reporting of solution strategies in their current practices. However, closer analysis highlighted that such reports took the form of "show and tell". The combined approaches described in this paper challenged their personal theories in action, made these problematic, and pressed them to rethink the role of reasoned discourse in mathematics classrooms. It was the practical set of pedagogical actions outlined on the CPF which scaffolded the teachers, and in turn their students, to deep engagement with both the discourse of inquiry and argumentation and use of a range of rich interrelated mathematical practices.

In summary, this paper highlights the importance of teachers developing a coherent conceptual framework of pedagogical strategies that can support the development of proficient mathematical practices within reasoned mathematical dialogue. The CPF provided a practical but flexible and adaptive tool which supported the establishment of the discourse of mathematical inquiry communities. The provision of time and space through flexible scheduling of three phases of change and movement towards inquiry communities and practices acknowledged the complexity of teaching and teacher change, and provided manageable steps for teachers to individually and collectively enact when changing the discourse practices in classrooms.

Too often professional development initiatives and action research type programmes are evaluated by teacher change–with the assumption that teacher change in the advocated direction, with adoption of advocated practices–is the measure of success or an end point. This study however, gave teachers the opportunity to assess the effectiveness of pedagogical change by providing a framework that directly addressed changes in student mathematical behaviour and thinking. The changes the teachers made were generative; they had learnt the skills of reflecting-in-action as they responsively attended to growing both student mathematical knowledge and its use in increasingly proficient ways.

## References


Ball, D., & Bass, H. (2003). Making mathematics reasonable in school. In J. Kilpatrick., G. Martin., & D. Schifter (Eds.), *A research companion to the principles and standards for school mathematics* (pp. 27-45). Reston, VA: National Council of Teachers of Mathematics.

Ball, D. L., & Forzani, F. M. (2007). What makes education research "Educational"? *Educational Researcher, 36*(9), 529-540.

Boaler, J. (2003a). Studying and capturing the case of the dance of agency. In N. Pateman, B. Dougherty, & J. Zilliox (Eds.), *Proceedings of the 27th annual conference of the International Group for the Psychology of Mathematics Education* (Vol. 1, pp. 3-16). Honolulu, HI: PME.

Boaler, J. (2003b). Exploring the nature of mathematical activity: Using theory, research and "working hypotheses" to broaden conceptions of mathematics knowing. *Educational Studies in Mathematics, 51*(1-2)), 3-21.

Forman, E. (1996). Learning mathematics as participation in classroom practice: implications of sociocultural theory for educational reform. In L. Steffe, P. Nesher, P. Cobb, G. Golding, & B. Greer (Eds.), *Theories of mathematical learning* (pp. 115-130). Mahwah, NJ: Lawrence Erlbaum Associates.

Franke, M. L., Kazemi, E., & Battey, D. (2007). Understanding teaching and classroom practice in mathematics. In F. K. Lester (Ed.), *Second handbook of research on mathematics teaching and learning* (Vol. 1, pp. 225-256). Charlotte, NC: NCTM.

Goos, M. (2004). Learning mathematics in a classroom community of inquiry. *Journal for Research in Mathematics Education, 35*(4), 258-291.

Hufferd-Ackles, K., Fuson, K. C., & Sherin, M. G. (2004). Describing levels and components of a math-talk learning community. *Journal for Research in Mathematics Education, 35*(2), 81-116.

Hunter, R. (2007). *Teacher developing communities of mathematical inquiry.* Unpublished doctoral dissertation, Massey University. Palmerston North.

Lave, J., & Wenger, E. (1991). *Situated learning: Legitimate peripheral participation.* New York: Cambridge University Press.

Lerman, S. (2001). Cultural discursive psychology: A sociocultural approach to studying the teaching and learning of mathematics. *Educational Studies in Mathematics, 46*, 87-113.

Ministry of Education. (2007). *The New Zealand curriculum*. Wellington: Learning Media.

RAND. (2003). *Mathematical proficiency for all students: Towards a strategic research and development program in mathematics education*. Santa Monica, CA: RAND

Sowder, J. T. (2007). The mathematical education and development of teachers. In F. K. Lester (Ed.), *Second handbook of research on mathematics teaching and learning* (Vol. 1, pp. 157-224). Charlotte, NC: NCTM.

Sullivan, P., Zevenbergen, R., & Mousley, J. (2002). Contexts in mathematics teaching: Snakes or ladders. In B. Barton, K. Irwin, M. Pfannkuch, & M. Thomas (Eds.), *Mathematics education in the South Pacific* (Proceedings of the 25th annual conference of the Mathematics Education Research Group of Australasia, pp 649-656). Sydney: MERGA.

Wood, T., & McNeal, B. (2003). Complexity in teaching and children's mathematical thinking. In N. L. Pateman, B. J. Dougherty, & J. Zilliox (Eds.). *Proceedings of the 27th annual conference of the International group for the Psychology of Mathematics Education* (Vol. 4, pp. 435-443). Honolulu, HI: PME.